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On Existence of Unique Solution by First Order System

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We consider the following system of simultaneous differential equations in terms of four components u, v, w and x and a vector functionθ.

$$u'' + lv + mw + nx = \lambda u$$
$$u - v'' + pw + qx = \mu v$$
$$mu + pv + iw' + r(x) = vw$$
$$nu + qv + rw - ix' = \eta x$$

whereu, v, w, x are functions of u, l, m, n, p, r are real valued continuous functions of t,λ, μ, v, η are parameters which may be real or complex,$t\varepsilon[a, b], i = \sqrt{-1}$, and dashes denote derivatives w.r.t. t.

Theorem: The system (1,1) of differential equations yields (admits) a unique solution

$$\theta(t) = (uvwx)^t(t)$$

satisfying the initial conditions

$$u^t(\alpha) = A_1$$
$$A^{(t)}(\alpha) = B_1$$
$$w(\alpha) = C_0$$

and $$x(\alpha) = D_{t^2}$$

where$A_1 B_t (x = 0,1), C_0, D_0$ are arbitrary constants (real) or complex) not all vanishing simultaneously, T denotes transpose (s) denotes sih derivatives w.r.t. t and $\alpha\varepsilon[a, b]$.

Proof. The system of differential equations (1.1) and set of initial conditions (1.2) may be alternatives written as:

$$u^u = -lv - mw - nx + \lambda u$$
$$v^u = lu + pw + qx - \mu v$$
$$w' = imu + ipv + irx - ivw'$$
$$x' = inu - iqv - irw + i\eta x$$
$$\big(u(\alpha), u(\alpha), v(\alpha), w(\alpha), x(\alpha)\big) = (A_0 A_1 B_0 B_1 C_0 D_0)$$

Futher for a vector$V$ letV^T denote the transpose of$V$ and

$$V^T = (uu'vv'wx)$$

where dashes denote derivatives w.r.t. t, then (1.3) and (1.4) have their respective equivalent form as:

$$V^T(t) = F(T)V(T)$$

and

$$V(\alpha) = (A_0 A_1 B_0 B_1 C_0 D_0)^T$$

Where

$$F(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \lambda & 0 & -\lambda & 0 & -m & -n \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -\mu & 0 & p & q \\ im & 0 & ip & 0 & iv & ir \\ -in & 0 & iq & 0 & -ir & i\eta \end{bmatrix}$$

Since V and F both are complex, hence we can write them as

$$V = V_1 + iV_2$$

and

$$F = F_1 + iF_2$$

WhereV_1, V_2 andF_1, F_2 are real matrices.
With the help of (1.6) we get from (1.5)

$$w'(t) = \begin{bmatrix} F_1 & -F_2 \\ F_2 & F_1 \end{bmatrix} w(t)$$

Where

$$w = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} w_0 = \begin{bmatrix} V_1(\alpha) \\ V_2(\alpha) \end{bmatrix} \qquad (1.7)$$

By Picard's theorem (Chapter 1 and 2 of ref 1, the expressions (1.7) yields a unique solution$\phi(t) = \left(u(t)v(t)w(t)tx(t)\right)^T$ depending analytically onλ.
This proves the theorem.

2. Construction of boundary condition vectors

We use the symbol

$$\varphi(a/x) = u\left(\frac{a}{x}\right)v\left(\frac{a}{x}\right)w\left(\frac{a}{x}\right))^T$$
$$= (uvw)^T(a,x)$$

$(a, x\varepsilon[a,b]$)to denote a solution of (1.1) satisfying the following set of conditions:

$$(u(a/x))_{x=\alpha} = u(a/a) = A_0$$
$$(u'(a/x))_{x=\alpha} = u'(a/a) = A_1$$
$$(v(a/x))_{x=\alpha} = v(a/x) = A_2$$

and

$$(w(a/x))_{x=\alpha} = w(a/x) = A_3$$

Where$A_s(s = 0,1,2,3)$ are arbitrary constants (real or complex).
Further, we use above symbols and

$$X_t(a/x) = u_t(a/x)v_t(a/x)w_t(a/x)^T$$

and

$$y_0(b/x) = \left(u_r\left(\frac{b}{x}\right)V_r\left(\frac{b}{x}\right)W_r\left(\frac{b}{x}\right)T(r = 1\ to\ 4)\right)$$

be vector solution of (1.1) satisfying the conditions at$x = a$ and$x = b$ respectively as:

$$u_1(a/a) = i, v_1(a/a) = 0, w_1(a/a) = 0, u_1'(a/a) = 0$$
$$u_2(a/a) = i, v_2(a/a) = i, w_2(a/a) = 0, u_2'(a/a) = 0$$
$$u_3(a/a) = 0, v_3(a/a) = i, w_3(a/a) = 0, u_3'(a/a) = 0$$
$$u_4(a/a) = 0, v_4(a/a) = 0, w_4(a/a) = 0, u_4'(a/a) = 0$$

to proof that$D(d) = Prs\ (d)$ is a real quantity.

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